ELECTRONIC APPENDIX

This is the Electronic Appendix to the article

Non-linear time-periodic (NLTP) models of the longitudinal flight dynamics of desert locusts *Schistocerca gregaria*

by

Graham K. Taylor & Rafal Zbikowski


Electronic appendices are refereed with the text; however, no attempt is made to impose a uniform editorial style on the electronic appendices.
APPENDIX. Error analysis and corrections to experimental data

Sources of measurement error can be either electrical or mechanical in origin. The former must be corrected prior to converting the voltage output of the force balance to force-moment units, whereas the latter must be corrected after. We will treat each in turn.

a) Electrical errors and corrections

i. Thermal drift corrections.

Changes in ambient temperature \( T_a \) and tunnel speed \( U \) cause the heat flux between the force balance and its surroundings to vary, leading to fluctuations in operating temperature \( T_b \). This changes the resistance of the strain gauges, causing a small voltage drift in the amplifier’s null output, defined as the output when no external force is applied. The magnitude of this drift is proportional to amplifier gain \( k \), but independent of excitation current \( I \). During the angle series experiments, ambient temperature rose steadily \( \Delta T_a \approx 1{\degree}C \), causing a measurable drift \( k\partial v/\partial t \) in the nulls (where \( v \) is a voltage), which we assumed to be linear through time and could therefore subtract from the data directly. During the speed series experiments, imposed changes in flow velocity \( U \) dominate the changes in heat flux, so we made a separate speed calibration where we measured the output of the unloaded balance over the range of speeds used in the experiments (8 different speeds, with 3 replicates at each). Linear regressions were fitted to these data, and the slopes of the regressions used to estimate \( k\partial v/\partial U \), which we used to correct the data.

ii. Electrical offset corrections.

Additive electrical offsets arise in two distinct ways. Firstly, manual zeroing of the amplifier is never perfect, so a small non-zero offset \( k v_0 \) will always be present, independent of the excitation current. Secondly, intrinsic differences in resistance across
the two sides of a Wheatstone bridge result in a small net resistance ($R_0$). This produces a further voltage offset ($kIR_0$) sensed only when excitation is supplied. After zeroing, the amplifier’s bridge balance facility was used to inject a small voltage $v_{trim}$ that brought the amplifier’s mean null output $V_{null} = k(v_0 + v_{trim} + IR_0)$ as near to zero as possible, and the remaining offset in $V_{null}$ was measured at the start of each experiment.

**iii. Electrical noise filtering**

Electrical noise ($\tilde{v}_{\text{noise}}$) is a particular issue with strain gauge measurements when the changes in resistance being measured are small. A combination of analogue and digital filtering was therefore used to clean the signal. The amplifier output was low-pass filtered at source using an analogue 4th order Chebyshev filter (-20dB cut-off frequency at 1kHz) to remove high-frequency white noise and to prevent aliasing during frequency domain analysis (Nyquist frequency 5kHz). This allowed us to use discrete Fourier transforms (DFT) to examine the spectral content of the signals. The power spectra display up to 16 discernable harmonics of wingbeat frequency (20Hz), so we truncated the spectra digitally at 340Hz to remove higher frequency noise.

Mains noise (at 50Hz) and its harmonics enter the signal through rectification of the amplifier power supply. The corresponding spectral peaks are very sharply defined, by virtue of the very high resolution (c. 0.08Hz) of the DFT, which results from the combination of high sampling frequency (10kHz) and long recording period (15s per sample). We therefore used a digital multi-notch filter to remove all spectral content within ±0.24Hz of the peak spectral component at 50Hz and each of its harmonics. These values were carefully selected to remove mains noise without impacting upon any harmonic of wingbeat frequency. The spectra were then converted back to the time domain by calculating their inverse DFT.

**iv. Corrected amplifier output**
In light of the errors discussed above, the total amplifier output is:

\[ V_{\text{tot}} = k \left( v_0 + v_{\text{trim}} + \tilde{v}_{\text{noise}} + \frac{\partial v}{\partial T_b} \left( \frac{\partial T_b}{\partial t} + \frac{\partial T_b}{\partial U} \right) \right) + I \left( R_0 + \frac{dR}{dF} \right), \]  \tag{A1} 

where \( dR/dF \) measures how the difference in resistance (\( R \)) across the Wheatstone bridge depends upon the total applied load (\( F \)). Rearranging Eq. A1, we may define:

\[ V_{\text{cor}} = kI \frac{dR}{dF} \approx V_{\text{tot}} - V_{\text{null}} - k\tilde{v}_{\text{noise}} - k\frac{\partial v}{\partial t} - k\frac{\partial v}{\partial U}, \]  \tag{A2} 

which prescribes how the corrections discussed above are to be implemented. Systematic error introduced by \( V_{\text{null}} \), \( k\frac{\partial v}{\partial t} \) and \( k\frac{\partial v}{\partial U} \) is removed simply by subtracting their contributions from \( V_{\text{tot}} \). Similarly, because \( \tilde{v}_{\text{noise}} \) combines additively with the signal, it is removed by the low-pass filtering and notch filtering applied in the discrete frequency domain.

Amplifier gain \( k \) and excitation current \( I \) may be assumed to remain constant throughout each experiment, but are likely to have changed slightly since balance calibration. Because \( k \) and \( I \) combine multiplicatively with \( dR/dF \) (Eq. A2), the equivalent voltage output (\( V_{\text{equiv}} \)) differs from \( V_{\text{cor}} \) by a constant factor. To calculate this factor, we shunted a precision resistor (32k\( \Omega \) or 49.9k\( \Omega \)) across one arm of each Wheatstone bridge to simulate a large applied strain, and compared the voltage output (\( V_{\text{shunt}} \)) obtained with its reference value (\( V'_{\text{shunt}} \)) during balance calibration. The equivalent voltage output to that which would have been obtained had the experiment been performed at the time of balance calibration is then simply:

\[ V_{\text{equiv}} = \frac{V'_{\text{shunt}}}{V_{\text{shunt}}} \cdot V_{\text{cor}}. \]  \tag{A3}
Having calculated $V_{\text{equiv}}$, we converted this fully corrected output to force-moment units using a static calibration of the balance, analysed as a GLM retaining significant terms up to third order in any one channel together with all significant second order interactions ($p<0.05$).

\[ b) \text{ Mechanical corrections} \]

The vector $\mathbf{F}$ containing the $(X, Z, M)$ components of the total load on the balance (resolved at the centre of gravity of the locust) is given by:

$$
\mathbf{F}(\alpha,U) = \mathbf{X}(\alpha,U) + \mathbf{D}_{\text{ref}} + \mathbf{W}_{\text{ref}} + \frac{\partial(\mathbf{D}_{\text{teth}} + \mathbf{D}_{\text{bal}})}{\partial \alpha} + \frac{\partial \mathbf{D}_{\text{teth}}}{\partial U} + \frac{\partial(\mathbf{W}_{\text{teth}} + \mathbf{W}_{\text{bal}})}{\partial \alpha},
$$  \hspace{1cm} (A4)

where $\mathbf{X}(\alpha,U)$ contains the forces and moments due to the locust itself. Extraneous aerodynamic drag acts on both the tether ($\mathbf{D}_{\text{teth}}$) and the exposed balance tip ($\mathbf{D}_{\text{bal}}$), but since the latter is already included in the measurement of $V_{\text{null}}$, the constant contribution $\mathbf{D}_{\text{ref}}$ refers only to the drag on the tether. $\mathbf{D}_{\text{bal}}$ is expected to be a quadratic function of $U$, but such changes in $\mathbf{D}_{\text{bal}}$ were already accounted for in the measurement of $k\partial \nu/\partial U$ above and are not corrected for again. Gravity enters the equation through the weight of the tether ($\mathbf{W}_{\text{teth}}$) and the self-weight of the balance ($\mathbf{W}_{\text{bal}}$), but since the latter is already included in the measurement of $V_{\text{null}}$, the constant contribution $\mathbf{W}_{\text{ref}}$ refers only to the added weight of the tether.

Because the mounting plate lay flush to the locust, and is unlikely to have contributed much drag, we were able to estimate the drag on the tether by modelling it as a circular cylinder (White, 1974). The estimated drag is of order 0.1mN (some 30 times smaller than the weight of the tether) so we may safely neglect both $\mathbf{D}_{\text{ref}}$ and $\partial \mathbf{D}_{\text{teth}}/\partial U$.

Rearranging Eq. A4, we therefore have:
\( X(\alpha, U) = F(\alpha, U) - W_{\text{ref}} - \frac{\partial W_{\text{teth}}}{\partial \alpha} - \left( \frac{\partial D}{\partial \alpha} + \frac{\partial W_{\text{bal}}}{\partial \alpha} \right) \) \hfill (A5)

where \( W_{\text{ref}} \) and \( \frac{\partial W_{\text{teth}}}{\partial \alpha} \) were calculated from the mass of the tether and the geometry of the balance. The bracketed terms \( \left( \frac{\partial D}{\partial \alpha} + \frac{\partial W_{\text{bal}}}{\partial \alpha} \right) \) were measured together directly in a separate angle calibration in which we measured the unloaded output of the balance over the complete range of angles used. We made five replicates at each angle and fitted a separate linear regression for each force-moment component to estimate \( \left( \frac{\partial D}{\partial \alpha} + \frac{\partial W_{\text{bal}}}{\partial \alpha} \right) \). After making these corrections, the weight of the locust was subtracted from Eq. A5 to give force recordings containing only the aerodynamic and inertial forces on the locust.