Correction to: ‘The entropic basis of collective behaviour’

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In our original manuscript [1], we made an erroneous claim about the general properties of Galton–Watson (GW) processes, based on a misreading of the literature. We claimed that

The Kesten–Stigum theorem states that for any GW process, the distribution of the number of nodes converges to an exponential distribution, conditional on the tree not becoming extinct.

In fact the Kesten–Stigum theorem [2] only states that the number of nodes will converge to some random variable, the distribution of which depends on the choice of offspring distribution. This random variable has a mean of \( z^h \), where \( z \) is the mean offspring per node and \( h \) is the number of generations.

Therefore, equation (3.2) in the original manuscript, and the results derived from it, hold in the reduced set of cases where the number of leaves in the tree of potential future choices converges to an exponential distribution. Though we cannot delimit all processes where this applies, there are two special cases with particular relevance to our manuscript:

— For a GW process where the number of offspring to any node is geometrically distributed (either single parameter or zero-modified), the number of leaves at generation \( h \) will also be geometrically distributed [3], and can be well approximated by an exponential distribution for large \( h \). The geometric distribution is the maximum entropy distribution over the integers with a known mean and therefore a natural choice when considering a process with a fixed mean number of offspring in the absence of other information.

— As stated in the original manuscript, the continuous time Yule process also converges to an exponential distribution at long time horizons.

Readers should therefore be aware that the derivations of collective and social behaviours described in the manuscript are valid within this bound. Alternative choices of GW or other branching process specifications will potentially result in different collective and individual behaviour predictions.

References