The seed capsule of Delosperma nakurense is a remarkable example of a natural hygromorph, which unfolds its protecting valves upon wetting to expose its seeds. The beautiful mechanism responsible for this motion is generated by a specialized organ based on an anisotropic cellular tissue filled with a highly swelling material. Inspired by this system, we study the mechanics of a diamond honeycomb internally pressurized by a fluid phase. Numerical homogenization by means of iterative finite-element (FE) simulations is adapted to the case of cellular materials filled with a variable pressure fluid phase. Like its biological counterpart, it is shown that the material architecture controls and guides the otherwise unspecific isotropic expansion of the fluid. Deformations up to twice the original dimensions can be achieved by simply setting the value of input pressure. In turn, these deformations cause a marked change of the honeycomb geometry and hence promote a stiffening of the material along the weak direction. To understand the mechanism further, we also developed a micromechanical model based on the Born model for crystal elasticity to find an explicit relation between honeycomb geometry, swelling eigenstrains and elastic properties. The micromechanical model is in good qualitative agreement with the FE simulations. Moreover, we also provide the force-stroke characteristics of a soft actuator based on the pressurized anisotropic honeycomb and show how the internal pressure has a nonlinear effect which can result in negative values of the in-plane Poisson’s ratio. As nature shows in the case of the D. nakurense seed capsule, cellular materials can be used not only as low-weight structural materials, but also as simple but convenient actuating materials.

1. Introduction

Natural hygromorphs are biological systems that can generate a specific actuation (i.e. displacement or force) as a response to a change in humidity occurring for example between day and night [1,2]. They are interesting examples for bioinspired materials design, as they typically operate without metabolic energy meaning that actuation is achieved through the arrangement or architecture of the underlying constituent materials (cellulose fibrils, hemicellulose, lignin, etc.) [3,4]. In other words, the material’s architecture of the actuating tissue controls and guides the otherwise unspecific isotropic swelling of the single components [5–7].

Natural hygromorphs can be found for example in the seed dispersal units of wild wheat awns [1], or in the scales of the pine-cone [8,9]. In each of these systems, the actuating organ is structured in a similar way to a bilayer, where two materials with different swelling properties are juxtaposed. These differences in swellabilities are a consequence of the microstructural organization of the reinforcing cellulose fibrils in the secondary cell wall. When water is absorbed into the swellable matrix polymers, volumetric expansion occurs mainly perpendicular to the stiff crystalline cellulose fibrils: the microfibril angle of the cellulose fibrils thus directs the swelling deformation of the cell [10,11]. At the macroscopic level of the bilayer, wetting/drying cycles correspond to an alternate bending/straightening motion, as predicted by the Timoshenko model of a two-material plate undergoing thermal deformations [11]. Another natural passive actuator exploiting the microscopic architecture of the cellulose fibrils is the awn of the stork’s bill [12]. The macroscopic
coiling movement appears to originate from a mechanically uniform cell layer made up of intrinsically coiling cells, rather than a spatial distribution of differently swelling materials [13]. In each cell, the cellulose helix is tilted with respect to the cell main axis: in a fully swollen state, the matrix exerts a pressure on the helix, which uncoils and aligns to the cell main axis; while drying, the helix tilts and tightens, thereby causing a macroscopic spiralling movement.

Cellulose microfibril orientation also controls the extent of forces generated during swelling. In the roots and xylem of higher plants, specialized tissues have evolved that are able to generate a contraction force (tensile stresses) on the surrounding tissues [14–16]. In these tissues, the cells possess thick walls with a lignified secondary cell wall having high microfibril angles (about 36°) and an innermost gelatinous layer (G-layer) consisting of axially oriented cellulose fibrils. One explanation for the development of tensile stresses comes from the idea that lateral swelling from the G-layer puts the cell walls under pressure. The resultant volume expansion occurs mainly along the cells cross-sectional short axis, with a net deformation of up to four times the original cell width. Hence, the swollen CIL exerts a pressure on the cells’ walls, giving rise to an overall shape change of the keel tissue (figure 1b,d). The physics of pressure generation by CIL’s swelling and its interplay with structures of the seed capsule to generate this reversible motion has been recently presented [19].

The shape of the cells in the swelling tissue also plays an important role in hygroscopic actuation systems. This was shown in a recent study on the hydro-actuated unfolding of the ice plant (Delosperma nakurense) seed capsule [18,19]. The seed capsule contains five seed chambers, separated by septa (figure 1). In the dry state (figure 1a), five valves close the chambers to prevent accidental seed dispersion, with the centre of each valve pushed against a corresponding septum. As liquid water comes into the tissues, the valves unfold (figure 1c), moving away from the septa, spanning an angular trajectory of about 150°. The specialized organ promoting this movement is the hygroscopic keel (highlighted in red in figure 1c), a prominent tissue attached to the centre of the inner valve surface, consisting of two halves which, in the dry state, pack onto the septum. Structural and chemical characterization of the keel tissue revealed a highly anisotropic honeycomb structure, with elliptical-shaped cells. The cell walls were lignified and lined with a ‘cellulosic inner layer’ (CIL) consisting of as-yet unidentified hygroscopic polysaccharides. In the dry state, the cells appear collapsed and closely packed together. During swelling, the CIL absorbs large amount of water. The result of volume expansion occurs mainly along the cells’ cross-sectional short axis, with a net deformation of up to four times the original cell width. Hence, the swollen CIL exerts a pressure on the cells’ walls, giving rise to an overall shape change of the keel tissue (figure 1b,d). The physics of pressure generation by CIL’s swelling and its interplay with structures of the seed capsule to generate this reversible motion has been recently presented [19].

Many studies have been conducted on the mechanical behaviour of regular cellular materials (e.g. [20–24]). These models estimate the macroscopic ‘continuum’ mechanical properties of a regular cellular material from microscopic parameters like nodal connectivity, cell shape, relative density and mechanical properties of the cell wall material [21,25–27]. Moreover, the influence of cell-size distribution, irregularity and point defects on the elastic and failure properties of honeycombs and foams has also been extensively studied [28–30]. In these studies, the mechanical properties of the cellular material are discussed by simulating ‘simple’ boundary value problems (uniaxial, biaxial or pure shearing loading). Despite the large amount of work being done on the mechanics of honeycombs, surprisingly, no systematic study has been conducted on internal loading conditions like hydrostatic pressurization on the honeycomb inner surface, relevant to understanding the actuation of the ice plants. One study by Niklas, which goes in this direction, provided a theoretical description of the effect of a turgor protoplast pressurizing a thick-walled plant cell on its effective stiffness [31]. Here, the cell was considered cylindrical and isolated, and no collective effect at the tissue level or cell shape influence was considered. Other studies [32,33] are concerned with the effect of a turgor pressure on the effective stiffness of foam solids like the parenchyma tissue, but their scope limits to isotropically shaped cells in the stretching-dominated regime. Some studies have investigated the influence of cell shape on the overall expansion of honeycombs, however, these considered only a small orthotropic expansion of the honeycomb walls rather than an internal pressurization [34]. No explicit investigation of microscopic swelling deformation at less than fully turgid pressures has been provided.

Internal pressurization in hollow structures has also awakened the interest of applied research fields such as soft robotics. In a recent study, Martinez et al. [35] combine elastomers with flexible inextensible paper reinforcements to produce a new class of soft pneumatic actuators. Here, the driving force for actuation comes from an external source of pneumatic pressure, while the desired actuation is controlled by the crease pattern and spatial arrangement of paper sheets embedded in an elastomer. Another study by Pagitz et al. [36] presents a novel concept for pressure-actuated cellular structures that is inspired by the nastic movement of plants.
They consider a composite plate-like cantilever formed by two arrays of prismatic cells with identical pentagonal and hexagonal cross sections. Differences in internal pressures for each array of cells, corresponds to an equilibrium configuration that resembles a circular arc. By changing the initial shape of the cells, non-circular arc shapes can also be achieved.

In the following, we will present a theoretical model for the assessment of the performance of a swellable honeycomb. This should be of some relevance for understanding the mechanical aspects related to the hygroscopic actuation in the ice plant seed capsules. Nonetheless, the model should also be general enough to give insight into the applicability of anisotropic honeycombs as possible candidates for pressure-driven actuators. The pressure may be generated from a swelling process (as in the keel tissue of the ice plant), or an external source (as in pneumatic soft robots). In both cases, notwithstanding the necessary simplifications introduced in the model, our aim is to give a deeper understanding of the mechanical performance of pressurized anisotropic honeycombs.

The actuation performance of the honeycomb will be investigated in terms of three complementary responses: the eigenstrains developed by the honeycomb upon a certain internal pressure (i.e. the magnitude of accessible motion), the forces generated at a given stroke, and its effective stiffness in a given pressurized configuration. In the following, we somewhat abuse the term eigenstrains since the effect of a pressure on such a hollow structure as the honeycomb will generally cause an elastic stress in its walls, and hence is not a stress-free transformation strain [37]. This notwithstanding, one considers the pressurizing agent as part of the investigated system (as in our contribution here), then the net elastic stresses are null and the macroscopic strain of the structure is stress-free in an averaged sense.

As a test case, we will consider a bi-dimensional anisotropic honeycomb consisting of diamond-shaped cells, inspired by the tissue of the ice plant keels. Our modelling approach, described in the Numerical model section, is based on a finite-element (FE) model comprising all relevant geometrical and mechanical parameters of the honeycomb. To calculate the quantities we are aiming at (eigenstrains, exerted forces and mechanical parameters of the honeycomb), we first simulate a hydrostatic pressurization, and get an equilibrated pressurized configuration characterized by its eigenstrains. From this configuration, we perform basic loading conditions of the structure such as uniaxial stretching and in-plane shearing, to get the full stiffness tensor of the pressurized honeycomb. These simulations are repeated in a parametric study where the effect of cell wall thickness and swelling pressure on the model’s behaviour is addressed. In §3.3, we will show how these results can be interpreted in terms of an analytical micromechanical model, and critically evaluate the prediction ability of our modelling strategy. In the final section, we will provide some conclusions and outlook for future work.

2. Numerical model

2.1. Shape of the unit cell and mechanical properties

2.1.1. Geometry of the investigated system

The honeycomb considered here is a diamond-shaped honeycomb resembling the ice plant keel tissue. The unit cell consists of two inclined beams, joined at an angle $\alpha = 28^\circ$. The regular lattice is obtained by shifting the unit cell along two lattice vectors at an angle $\pm \alpha/2$ to the horizontal. In contrast to a hexagonal honeycomb, each joint of the infinite lattice connects four inclined members (figure 2a).

2.1.2. Finite-element model

In the dry state, the ice plant keel tissue is completely collapsed, with barely no cell lumens observable. In the swollen state, the cells’ walls are pushed apart by the swelling of the CIL inside the lumens (figure 1). Based on the volumetric expansion that it undergoes, in the swollen state, the material in the lumen can contain up to 95% water. Although direct mechanical measurements of the CIL have proved impractical, it can be inferred that the lumen’s elastic modulus is negligible, when compared to the much stiffer lignified walls.

Following this abstraction (figure 2b), the unit cell of the FE model measures $X_0 = 80$ mm by $Y_0 = 20$ mm and comprises a structural domain coinciding with one full diamond-shaped honeycomb cell and a fluid domain occupying the remaining space; to be close to the keel’s geometry, the walls are modelled as slender beams of length $l = 41$ mm and thickness $t = 3$ mm so that $1/l > 10$. The resulting honeycomb angle measures $28^\circ$; with such an anisotropic shape, care has been taken to avoid excessive bulkiness of the joints. These are built as squares of side $\sqrt{2}s$ ($s = 2.5$ mm) (red dotted area) in order to make their

---

Figure 2. Diamond honeycomb geometry. Diamond honeycomb geometry results from periodic repetition of the unit cell (dashed box) along lattice vectors at an angle $\pm \alpha/2$ to the horizontal so that each joint connects four inclined members (a). This geometry has been imported in the FE model unit cell as a fluid-filled honeycomb with finite thickness walls (b). The FE honeycomb has slender walls ($r = 3$, $f = 41$) and strong anisotropy ($X_0/Y_0 = 4 : 1$). The joints (red dotted online and thick dotted grey in print) are built as squares of side $\sqrt{2}s$ ($s = 2.5$ mm). (Online version in colour.)
rigidity independent from the direction of the loading. The walls are considered to be linearly elastic with a Young modulus (1 GPa) and Poisson’s ratio (0.3), according to values reported for cell wall transverse modulus of spruce wood in moist conditions [6]. The lumen is modelled as a fluid cavity subjected to a constant pressure $p$. The geometry, material constitutive equation, boundary conditions and discretization have been implemented in the commercial FE software ABAQUS v. 6.12 (Dassault Systèmes Simulia Corp.). In particular, the structural domain was discretized with 1040 linear, two-dimensional, plane stress, reduced integration continuum elements (ABAQUS v. 6.12 element library: CPS4R), with four elements across the wall’s thickness to precisely capture the stress gradient. The ‘enhanced hourglass control’ option has been used to avoid deformation artefacts at the fluid–structure boundary. Mesh size independence has been verified by using a superfine 16 704 elements mesh (16 elements across the wall thickness), resulting in predictions of honeycomb expansion deviating by less than 1%.

2.2. Eigenstrains in a finite-sized honeycomb
To assess the tissue strain as a function of swelling pressure, a system of $5 \times 10$ cells has been considered. Here, the fluid elements have been subjected to a hydrostatic pressure, leaving the external boundaries free, and symmetry conditions were applied to avoid rigid body motion. Upon pressurization, the system undergoes an anisotropic deformation (figure 3a–c). Along the $y$-direction, which is the soft direction of the honeycomb, the system expands greatly, even at very low pressure. On the other hand, in the $x$-direction the system shrinks slightly, and then expands again at higher pressures. Looking at the single cells the deformation is characterized by two regimes. At lower pressure, the

Figure 3. Deformation of an anisotropic honeycomb with isotropic linear elastic walls ($E = 1$ GPa, $\nu = 0.3$) under internal pressure. (a) Initial configuration at zero pressure. (b) At lower pressure ($p = 0.3$ MPa), the cells volume is maximized by bending deformation of the walls and localized distortion of the joints with an increase in the honeycomb angle (bending regime). (c) As the internal pressure increases ($p = 7.5$ MPa), the further volume increase can be accommodated only by stretching the walls (stretching regime). Swelling eigenstrains, (d) in the $x$-direction and (e) in the $y$-direction as a function of inner pressure. The swelling eigenstrains change rapidly at low pressures, where the honeycomb deforms mainly by bending and joint opening. Along the $y$-direction there is up to a threefold expansion, while the $x$ dimension shrinks. The dotted line ($p = 2.25$ MPa) highlights the transition from a bending-dominated (a) to a stretching-dominated (b) regime. At higher pressures ($p > 10$), the loading of the honeycomb becomes unstable and the strains diverge. The reason for this is highlighted in (f) which shows the influence of internal pressure on wall stretching in a square vessel using a modified Laplace Law that includes the effect of the Poisson’s ratio of the cell walls. At low pressures, the walls stretch linearly with the pressure, however as the pressure increases, a non-zero Poisson’s ratio leads to wall thinning. The pressure has a positive feedback on the strain in the walls and at a certain (finite) pressure the load is unbalanced and the strain diverges. As a comparison, a material with zero Poisson’s ratio does not experience this type of unbalanced loading at least for reasonable values of strains. (Online version in colour.)
honeycomb walls deform mostly in bending; their deformed shape is sigmoidal, with localized rotation at the joints that causes an opening of the honeycomb angle and a net increase in the fluid volume. As the pressure increases, the honeycomb angle approaches its maximum value of 90°, while the walls become stretched. At very high pressure, any further volume increase happens at the expense of significant wall stretching.

2.3. Eigenstrains in an equivalent periodic system

The specimen size, relative to the cell size, is known to influence the assessment of mechanical properties like stiffness and strength. Such scale effects depend on the loading type (shear, uniaxial), but usually become negligible in specimens larger than five repetitions of the unit cell [38]. In order to calculate the eigenstrains and properties of the homogeneous honeycomb, we investigated the system of a single unit with periodic boundary conditions. We start with the diamond structure (figure 2) and add a constant fluid pressure \( p \) into the lumina, whereby the problem is treated in two dimensions. \( x_0 \) and \( y_0 \) are the horizontal and vertical dimensions of the diamond before deformation and \( X \) and \( Y \) after some deformation. Periodic boundary conditions imply that \( X \) and \( Y \) are fixed and only some deformation of the walls may occur due to compressibility of the wall material (\( \nu < 0.5 \)). Hence, for each value of \( X \) and \( Y \), an FE simulation can be used to determine the equilibrium shape of the diamond and the associated elastic energy \( E_{el}(X,Y) \). For symmetry reasons, we do not expect shear to occur spontaneously in the system (at least not with positive internal pressures). Therefore, the total energy change of the system due to deformation is

\[
E_{tot} = E_{el}(X, Y) - 1/2p\,(XY - X_0Y_0). \tag{2.1}
\]

The first term is the elastic energy stored in the walls which is counteracted by the second term, the work done by the internal pressure. Minimizing this expression for \( X \) and \( Y \) should give the shape corresponding to the mechanical equilibrium. In practical terms, this means numerical calculation of \( E_{tot} \) for a range of \((X, Y)\)-values and then searching for the one that minimizes \( E_{tot}\). The swelling strain referred in the following as the eigenstrain is then simply:

\[
e = \begin{pmatrix}
\frac{\bar{X}}{X_0} - 1 & 0 \\
0 & \frac{\bar{Y}}{Y_0} - 1
\end{pmatrix}, \tag{2.2}
\]

where both \( \bar{X} \) and \( \bar{Y} \) are the values of \( X \) and \( Y \) that minimize the total energy, which are thus functions of the internal pressure \( p \) (see the electronic supplementary material).

2.4. Finite-element homogenization and parametric study

The effective elastic properties of the pressurized honeycomb in its equilibrium shape can be calculated via a numerical homogenization. A bi-dimensional, two-plane symmetric, plane stress system as the anisotropic honeycomb considered here will generally show orthotropic elastic properties described by a stiffness tensor of the kind

\[
C = \begin{bmatrix}
C_{11} & C_{12} & 0 \\
C_{12} & C_{22} & 0 \\
0 & 0 & C_{66}
\end{bmatrix}. \tag{2.3}
\]

Basic loading states were simulated deforming the relaxed system (in equilibrium with the fluid pressure) with uniaxial stretching/shrinking in directions 1 and 2 and an in-plane 12 shearing. Then the corresponding effective stiffness component is calculated as the ratio between the average stress on the boundary and the applied probing strain [39].

Pressure equilibration and subsequent elastic properties homogenization were repeated in a parametric study spanning several values of \( p \) (from 0 to 9 MPa) and \( E \) (from 1 to 9 GPa).

3. Results

3.1. Baseline finite-element model

As for a square lattice, the diamond honeycomb considered here is a bending-dominated lattice, meaning that the main deformation modes and hence the apparent stiffness depends on the bending rigidity of the beams. In the baseline model, the wall stiffness is set to 1 GPa and the inner pressure varies up to 13 MPa. As a consequence, already at pressures lower than 1 MPa the honeycomb is able to double its original dimension along the y-direction (figure 3\( \mu \)). This is due to the bending-dominated deformation and localized deformation at the joints already described for the finite-size system (figure 3). At the same time, the structure shrinks only slightly along the x-direction. Around \( p = 2.25 \) MPa, the horizontal strain shows a minimum and the vertical one an inflection point. From this point onwards, as pressure increases, we observe that both eigenstrains increase, although with a reduced slope. Here, the joints are already completely opened and any further increase of the fluid volume must be accommodated by stretching of the honeycomb walls. At very high pressures (\( p > 10 \) MPa), the eigenstrains increase more than linearly and diverge. The reason for this divergence is due to the properties of the cell wall material rather than the structure and can be understood by the following simplified example.

In the stretching-dominated regime, all cells are fully expanded and have a square shape (figure 3c). Since the cells are all equal and the internal pressure is the same, we can study only one cell with side length \( l \) and wall thickness \( t/2 \) (where \( l \) and \( t \) are the length and thickness of the honeycomb beams). This cell is a pressure vessel; for a cylindrical pressure vessel, the longitudinal stress in the wall is given by the Laplace Law

\[
\sigma_L = \frac{p}{t}, \tag{3.1}
\]

where \( R \) is the radius of the vessel, and \( T \) is the wall thickness. In our case, as the pressure increases, the cells' volume increases while the walls become thinner according to our choice of material Poisson's ratio (\( \nu = 0.3 \)). Equation (3.1) can be expressed in terms of \( \sigma_L \) assuming plane stress and using the constitutive relations \( \sigma_1 = (E/(1-\nu^2))(e_{11} + \nu e_{22}) \), \( \sigma_2 = (E/(1-\nu^2))(e_{22} + \nu e_{11}) \). Assuming \( \sigma_1 = \sigma \leq \sigma_L \), we can substitute \( \sigma_2 = 0 \); also, given the square geometry of the cell, in the pressurized state \( R = l/(\sqrt{2})(1 + e_L) \) and \( T = (l/2)(1 + e_T) \). With these assumptions, equation (3.1) can be rearranged to become

\[
E_{el} = p\sqrt{2}\frac{f(l + e_L)}{f(l + \nu e_L)}, \tag{3.2}
\]

which states that the walls' stretching in the pressurized state depends on the wall's aspect ratio, the material properties and
on the internal pressure. Since in our honeycomb $I/t \approx 10$, we can solve equation (3.2) as a function of the pressure. It is straightforward to verify that the wall strain increases linearly at low pressures and diverges at a finite value of pressure (figure 3f). This is exactly what we observe in our FE simulations at the transition from the stretching-dominated regime to even higher pressures. In such cases, it was not possible to find a stable equilibrium configuration (in FE) past a certain value of pressure. Materials with $\nu = 0$ do not experience this instability (at least for reasonable walls strains), which means that the third regime is material (rather than structure) dependent and has to be considered in real applications.

Hence we can describe the swelling or actuation behaviour of the structure as being characterized by a bending-dominated regime where the structure expands more sensitively to fluid pressure and a stretching-dominated regime, where the structure is less sensitive. The maximum expansion and the relative importance of the two regimes depend on the geometry of the honeycomb: the more anisotropic the cells making up the honeycomb, the larger the maximum expansion, with a narrower bending regime and high pressure-stroke sensitivity. Conversely, a pressurized regular square lattice would show no bending-dominated regime, small expansion and poor sensitivity.

### 3.2. Parametric study

The results from the parametric study on the FE model are summarized in figure 4 in the form of eigenstrains and effective stiffness maps along the stiff ($e_y$, $C_{11}$) and soft axes ($e_x$, $C_{22}$) of the honeycomb as a function of pressure ($0 < p < 9$ MPa) and wall Young’s modulus ($1 < E < 9$ GPa), whereas the initial geometry of the model is preserved (walls aspect ratio $I/t$ and honeycomb angle $a_0$ stay constant). At this level of pressure, $e_y$ is always negative and small (some percentage shrinkage), whereas $e_x$ is everywhere positive and very large (up to threefold expansion). Both the $e_x$ and $e_y$ maps (figure 4a,b) show a similar distribution across the parameter space: their absolute value increases from the top-left corner (stiff honeycomb walls, low internal pressure) to the bottom-right corner (high pressure and soft walls). Nonetheless, while the $e_x$ is monotonic throughout the whole parameter space, the $e_y$ map shows a peak that does not coincide with the bottom-right corner. Here, at low wall stiffnesses (approx. 1 GPa) and high inner pressures ($p > 5$ MPa) $e_y$ increases again. This behaviour derives from the deformation mechanism of the walls which is a combination of large bending at low pressures and smaller amounts of stretching at high pressures. Shifting from soft to stiffer walls allows the effect of the inner pressure to be tuned: at low wall stiffnesses ($E = 1 – 2$ GPa), only a small pressure jump (from 0 to 2 MPa) suffices to increase the eigenstrains abruptly, whereas for higher walls moduli, this effect is much more gradual.

As for the effective stiffness (that is, the apparent elastic modulus of the honeycomb when subjected to a load, where subscripts 1, 2 refer to loading directions $x$, $y$), there is a monotonic (although nonlinear) proportionality between the walls modulus and $C_{11}$, $C_{22}$ (figure 4c,d). Somewhat counterintuitively, a pressure increase stiffens the weak direction ($C_{22}$ increases towards the right figure 4d) but also softens the strong one ($C_{11}$ increases towards the left, figure 4c). Generally, pressure acting inside the cells would be expected to stiffen the structure, in contrast to what is observed here for $C_{11}$. It is likely that this softening effect is geometry-related. As pressure deforms the honeycomb, almost axially oriented walls are rotated away from the $x$-axis, which places them in bending thus reducing the overall structural stiffness in the $x$-direction.
To investigate this idea, we try to understand this evidence in terms of a simpler analytical model governed by explicit geometrical parameters that fully characterize the honeycomb.

3.3. Born lattice model

A simpler equivalent mechanical model of the diamond honeycomb is an oblique two-dimensional lattice with centres sitting in the honeycomb joints. The deformation state of this lattice is fully identified by the value of the lattice angle \( \alpha \) and the point-to-point strain \( e_s \), so that the lattice strains \( e_{xy} , e_{yx} \) along the principal directions are obtained by a simple projection. Following other researchers [40,41], we describe the lattice elasticity by means of the Born model. Here, each material point is connected only to its nearest neighbours by two kinds of springs: a longitudinal spring \( K_l \) and a transverse one \( K_t \). The non-zero transverse stiffness \( K_t \) is needed to avoid the so-called uniconstant elasticity theory governed by ‘Cauchy relations’. For an isotropic medium, the Cauchy relations imply a Poisson’s ratio of one-quarter for all materials described by the model [42].

As already pointed out by Fratzl & Penrose [43], this is a consequence of considering just central forces for a crystal, which clearly does not hold for real crystals or general lattice materials. Since the lattice geometry replicates the continuum model used in the FE simulations, we assign to the longitudinal and transverse spring constants a value reflecting their axial and bending rigidity, which changes as a function of the angle at which they are loaded (see the electronic supplementary material)

\[
K_l = E l \frac{1}{C_1(\alpha)}, \quad K_t = E t \frac{1}{C_2(\alpha)}. \tag{3.3}
\]

The fluid-related term is simply written as the work done by the fluid from the undeformed normalized volume \( V_0 = 1/2 \sin(\alpha_0) \) to a generic deformed one \( V = 1/2 (1 + 2e_\|)\sin(\alpha) \). The pressure in general is not restricted to a specific form, but following the FE study, we choose a constant value (isobaric) which is independent of the volume of the single cell. In passing, we also note that different forms of pressure terms could also be implemented in the model to capture its potentially different physical origins. For example, an osmotic pressure could be included and will, in general, be dependent on the volume of the solute–solvent mixture, while a fluid confined to the cells will undergo an isothermal transformation.

For such a system, we can write the internal potential

\[
W = W_{\text{stretching}} + W_{\text{bending}} + W_{\text{pressure}}, \tag{3.4}
\]

where

\[
W_{\text{stretching}} = K_l (\alpha) \left[ \frac{e - e_0}{\sin(\alpha_0)} \right]^2, \\
W_{\text{bending}} = K_t (\alpha) \left[ \frac{\sin((\alpha - \alpha_0)/2)}{\sin(\alpha_0)} \right]^2, \\
W_{\text{pressure}} = -p \frac{V - V_0}{V_0},
\]

and

\[
e_s = \left[ \frac{(1 + e_\|) \cos(\alpha_0/2) - 1}{\cos(\alpha_0/2)} \right] \sin(\alpha),
\]

\[
e_{xy} = \left[ \frac{(1 + e_\|) \sin(\alpha_0/2) - 1}{\sin(\alpha_0/2)} \right] \cos(\alpha).
\]

For \( E = 1 \) GPa, \( d e_\|/dp = 0 \) falls at \( p = 3.03 \) MPa which, although not being exactly the same, is still quite close to the value observed in the FE simulations \( p = 2.25 \) MPa, considering the relatively broad minimum observed for both simulations (see figures 3d and 7a for a direct comparison).

The effective stiffness tensor components instead are found via the following:

\[
C_{11} = \frac{\partial^2 W}{\partial e_\|^2}, \quad C_{22} = \frac{\partial^2 W}{\partial e_{xy}^2}, \quad C_{12} = \frac{\partial^2 W}{\partial e_\| \partial e_{xy}},
\]

and

\[
C_{66} = \frac{\partial^2 W}{\partial e_{xy}^2}.
\]

The reference configuration in this case is the honeycomb’s pressurized state where \( \alpha = \alpha_p, e_s = e_p \).
3.4. Deformation modes of a Born lattice under pressure

The internal potential is described by the strain energy density of the mechanical lattice–fluid system. In figure 5, the energy density ‘landscape’ of the lattice is reported as a function of the principal lattice strains $e_x, e_y$ (mapping $(\alpha, \epsilon) \rightarrow (e_x, e_y)$), where the internal pressure is set to zero (for convenience, the energy density value is normalized by the value of wall stiffness $E$). The $y$-strain spans a broader range than the $x$-strain since this is the weaker direction. As expected, the steepest path runs across the first and the third quadrant where a biaxial deformation ($(e_x > 0, e_y > 0)$ and $(e_x < 0, e_y < 0)$) causes the lattice beams to deform axially, corresponding to a volumetric deformation of the lattice. Conversely, the lowest isolines are found for $(e_x < 0, e_y > 0)$ a pure deviatoric deformation of the lattice. In this narrow valley, the energy minimum is found (for $e_x = 0, e_y = 0$).

Upon pressurizing the system, the landscape changes considerably. In figure 5, the normalized energy contours are reported for increasing values of the normalized internal pressure $p^*$ ($p^* = p/E$).

There are two effects that can be observed. Firstly, the energy minimum migrates from the left area $(e_x < 0, e_y > 0)$ to the upper right area $(e_x > 0, e_y > 0)$ of the deformation space. The location of the minimum in each plot gives the deformation state of the lattice as it goes from the unloaded to the pressurized state. Indeed, the corresponding strains compare well with the swelling eigenstrains in the FE simulations (figure 4). Secondly, the shape of the isolines changes from an arc to a closed rounded one, which is a hint for a structural transformation of the system. This is confirmed by the angular deformation of the lattice for increasing normalized pressures (figure 6), where the lattice angle approaches $90^\circ$ (isotropic geometry).

3.5. Finite-element and Born model comparison

Considering several values of fluid pressure and Young’s modulus of the walls, actuation maps similar to those of figure 4 can be created. For clarity, each tensor component is normalized by the Born modulus of the material $E$ ($C_{ii}^B = C_{ii}/E$) and presented as function of normalized pressure $p^*$ (figure 7): by doing this, all data collapse onto a single master curve. Tensorial quantities in figure 7 are calculated by evaluating equation (3.7) with finite differences. Their ‘true’ value is found for an infinitesimal probing strain $(e_x, e_y)$ ($\gamma_{xy}$). Large (>10%) probing strains will cause the honeycomb geometry to distort, introducing geometric nonlinearities [44]. Thus, the stiffness tensor components reported in figure 7 are evaluated at the pressurized state $(\alpha_0, \epsilon_0)$ of the Born lattice applying the same probing strain $e_x = e_y = 1\%$ used for the FE parametric study. This is important to emphasize, since the strain energy density $W$ depends on the choice of the reference configuration.

From the master curves, there is a good qualitative agreement between the FE and Born models for the whole extent of the parametric space considered (figure 7). The direct components $C_{11}$ and $C_{22}$ (figure 7c,d) experience, respectively, a rapid increase/decrease for a slight increase of $p^*$ ($0.005$). This is due to the process already described: as the pressure increases, the lattice switches rapidly to a more isotropic geometry, strengthening the soft direction $y$ at the expense of the $x$-direction. At higher pressures ($p^* > 0.005$), the FE model predicts a softening of $C_{11}$ and $C_{22}$ which is not observed in the Born model. A further increase in pressure will not affect the honeycomb geometry (which already approaches a square (figure 6)) so that the Born model prediction of these components tend to a constant value. Therefore, we conclude that the softening effect observed only in the FE simulations is due to the walls strain divergence at high pressure due to Poisson’s effect introduced in §3.1.

The cross stiffness component $C_{12}$ (figure 7e) is characterized by a fast increase at low pressure (consistent with strong geometry change in the bending-dominated regime), but then decreases linearly at higher pressures, which (for the reasons just introduced) cannot be related to geometry changes. Moreover, since this behaviour is observed in both the FE and Born models, it cannot be ascribed to the aforementioned Poisson’s effect. This peculiar behaviour can be explained if we recall that $C_{12}$ measures the response of the material transversally to the probing strain direction. The transverse reaction force will be partially compensated by the internal pressure, thereby causing the observed inverse linear dependence.

The $C_{66}$ component evidences a poor agreement between the two models. This is not too surprising, since the parametrization of the polynomials $c_1$ and $c_2$ implies only relative displacement between the joints (see §3.3). To improve the prediction for the $C_{66}$ component one could introduce a localized joint rotation as an additional microstructural parameter. This is equivalent to a move from a classical continuum mechanics theory (where the strain state depends only on the displacement from the reference to the deformed configuration) to a Cosserat continuum theory (where the strain state depends also on microscopic rotations) [26].

3.6. Honeycomb-based actuator

Figure 8 shows the performance characteristics of the pressurized honeycomb described earlier $(\alpha_0 = 28^\circ, 1/l > 10, E = 1$ GPa) when used as an actuator. Reported is the force exerted as a function of the stroke (i.e. actuation stress versus strains) for several values of the pressure fed in the
system. Following Zupan [45], we describe the actuator performance in terms of two complementary normalized attributes. The maximum stroke divided by the actuator length parallel to this stroke is the actuator strain (or normalized stroke), and the maximum generated force divided by its cross-section perpendicular to the stroke is the actuator stress (or normalized force). In this honeycomb actuator, the energy required to produce mechanical work is stored in the fluid. An external compressor is ideally connected to the volume enclosed by the honeycomb walls and a flexible membrane sealing the top and bottom surfaces (not shown). In figure 8a, b, two working configurations of the honeycomb actuator are depicted, where it is respectively used to produce force along the y- and x-directions. In both cases, the transversal direction is free to deform. In the third configuration (figure 8c), the actuator is generating force along the x-direction, being constrained along the y-direction. Each graph represents a family of curves depicting the output stress at different pressure inputs: the actuator performance changes drastically at different levels of energy source. As expected, the maximum stroke (where the output curve meets the zero stress axis) coincides with the actuation eigenstrains.

Interestingly, the force output along the weak direction y is non-monotonic with the stroke. Obviously, it starts from a
zero-stroke value close to the set value of the pressure (which measures the amount of energy fed in the system) but, as the stroke increases, it firstly increases to an upper limit and then decreases till eventually it reaches zero. The stress produced is bigger than the pressure input into the system. This is not contradictory, but is a sign that the actuator works in different configurations as the stroke changes. At low strokes ($dy/dy_0 < 1$), the expanding fluid has to overcome just a small energy barrier (walls bending). The pressure is constant but since the fluid volume increases, the total energy available to produce mechanical work increases with the stroke. At higher strokes, the honeycomb walls start deforming in stretching, hence the energy cost is higher and the force generated lower, until it eventually reaches zero. In this configuration, the system capitalizes on the big angular deformation of the walls to produce high strains at moderate stresses.

Conversely, when the actuator is used to produce force along the strong $x$-direction (figure 8b), the stress output is high and the strokes are low. More importantly the stress generated is negative: the actuator ‘pulls’. Moreover, unlike the force output along the weak direction $y$, increasing the input pressure does not always mean a higher tensile stress. At a certain pressure ($p > 2$ here), the stretching of the walls becomes significant and partially compensates the contraction along the $x$-direction. Hence the actuator can be fine-tuned, to produce different tensile stresses at several strokes.

Constraining the $y$-direction (figure 8c) causes the actuator to produce a compressive stress, which is smaller, in absolute terms, than in the unconstrained case: since the honeycomb angle cannot open, the only possible deformation mechanism is due to walls stretching, which is rather energetically costly. Here, the actuator works in a low-strain, low-stress manner. Such a behaviour is reminiscent of the lever-arm principle used by wood cells to generate tensile or compressive stresses [10]: if the microfibril angle (equivalent to half of the honeycomb angle here) is less than 45° the only way to swell the cell is to shorten it along the longitudinal direction (the strong direction here).

As the pressure increases, $\alpha_p$ becomes larger (figure 6) and $\nu_{21}$ quickly goes to unity. The solid line in figure 9 shows the
prediction given by the pressurized honeycomb model subjected to a strain \( \varepsilon_2 = 1\% \). Obviously, the two curves coincide at \( p = 0 \) and are close for small values of pressure. Our model predicts larger values at negative pressures. At positive pressures however, the pressurized honeycomb prediction diverges to even more negative values. Since the applied probing deformation is positive, the available volume in the honeycomb increases and the fluid (which is kept at constant pressure) also expands in the transverse direction. As a result, the sign of Poisson’s ratio becomes negative. This effect is qualitatively similar to the long-term positive cross-sectional strains that a longitudinally prestretched rubber-like polymer specimen experiences when it is immersed in a fluid [46]. In summary, a pressurizing fluid put in the honeycomb can act as a free parameter to drastically change its Poisson’s ratio even to negative values. This can be useful in a number of applications, as auxetic materials can show enhanced shear modulus compared to positive Poisson’s ratio materials, and can be efficient materials for sandwich cores [47].

4. Conclusion

Cellular materials and honeycombs have long been among smart alternatives to classical bulk-materials, which give tunable material properties via control of the underlying pore or cell architecture. Although known for improving the range of applications of classical materials with respect to many applications—load-bearing, crash energy absorbers, thermal insulators—they could potentially also be used in the field of linear actuators. In the case of honeycombs subjected to inner pressure, we showed how their intrinsic characteristics (anisotropy, material properties, input pressure) could greatly simplify the design process for a linear actuator and the choice of the relevant parameters. The case of \( D. \text{ natarense} \), seed capsule provides an ideal natural system using this concept, optimized to work in a high-stroke, low-force configuration. Pressurized honeycombs could prove to be very versatile: similar systems can be implemented with different values of pressure and Young’s modulus to scale up and generate bigger forces. In addition, the very same honeycomb can be used in different working configurations to generate either compression or tension.

In this paper, we mainly discussed the influence of pressure and wall properties on eigenstrains, generated forces and effective stiffness. The next step would be to apply the same approach (analytical modelling via Born model and validation with FE simulations) to explore the role of architecture on the behaviour of a material under pressurization: a more complex material architecture—as for asymmetric, aperiodic frameworks and non-convex cells—could greatly expand the actuation capabilities upon pressurization. From a methodological viewpoint, this work tackles all the relevant mechanical aspects of a pressurized honeycomb, since the simplified lattice spring model predicts the FE simulations, while being beneficially easier to implement. In addition, a theoretical basis was provided to compare the finite-size and the periodic FE systems. As a future development of this study, an environmental sensitive pressure source should be considered, as in the case of a superabsorbent hydrogel confined in the honeycomb cells, which is a closer biomimetic embodiment of the natural system. Also, a theoretical description of out of plane effects deriving from frustrated or hindered volumetric expansion or tapering of the honeycomb walls along the cell’s axis could expand the range of the available motion of this planar symmetric system to the full three-dimensional space.

Acknowledgements. The authors thank Luca Bertinetti, Khashayar Razghandi, Matt Harrington and Ingo Burgert for extensive discussions. Funding statement. We acknowledge the Humboldt Foundation for supporting the visit of Y.J.M.B. to Potsdam through the Gay-Lussac-Humboldt Award and support through the Leibniz prize of P.F. running under DFG contract no. FR2190/4-1.

References

Correction

Cite this article: Guiducci L, Fratzl P, Bréchet YJM, Dunlop JWC. 2014 Pressurized honeycombs as soft-actuators: a theoretical study. J. R. Soc. Interface 11: 20141031.
http://dx.doi.org/10.1098/rsif.2014.1031

Pressurized honeycombs as soft-actuators: a theoretical study

Lorenzo Guiducci, Peter Fratzl, Yves J. M. Bréchet and John W. C. Dunlop

J. R. Soc. Interface 11, 20140458 (6 September 2014; Published online 25 June 2014) (doi:10.1098/rsif.2014.0458)

The authors discovered an error in the numerical code used to produce the force-stroke plots shown in figure 8. This error lead to non-monotonic behav-iour in both figure 8a,b of the published paper. This error has been fixed and was extensively tested with both the Born model and Finite-Element models. The corrected figure calculated using the Born Model is shown below. Note, all force-stroke curves in the corrected diagram are monotonic. This error also means that our interpretation of the reason for the non-monotonic behaviour of the force-stroke curves in the discussion of the old figures at the start of page 10 (first two paragraphs) is also erroneous. We originally thought this behaviour was due to the change in honeycomb configuration with swelling.

Figure 8. Working configurations and force developed by a honeycomb actuator (calculated by the Born Model). A pressurized honeycomb (grey) can be used as a linear actuator to displace an external load (dark cube). At least, three configurations can be thought of: (a) applying a moderate pushing force at high strokes, (b) applying a high pulling force at low strokes and (c) applying a low pushing force at very low strokes. Three graphs (bottom) show the working characteristics for different pressures fed in the honeycomb.