Crickets use so-called clavate hairs to sense (gravitational) acceleration to obtain information on their orientation. Inspired by this clavate hair system, a one-axis biomimetic accelerometer has been developed and fabricated using surface micromachining and SU-8 lithography. An analytical model is presented for the design of the accelerometer, and guidelines are derived to reduce responsivity due to flow-induced contributions to the accelerometer’s output. Measurements show that this microelectromechanical systems (MEMS) hair-based accelerometer has a resonance frequency of 320 Hz, a detection threshold of 0.10 ms$^{-2}$ and a dynamic range of more than 35 dB.

The accelerometer exhibits a clear directional response to external accelerations and a low responsivity to airflow. Further, the accelerometer’s physical limits with respect to noise levels are addressed and the possibility for short-term adaptation of the sensor to the environment is discussed.

1. Introduction

In biology, mechanosensors, equipped with differing hair-like structures for signal pick-up, are sensitive to a variety of physical quantities such as acceleration, flow, rotational rate, balancing and IR-light [1,2]. As an example, crickets have various types of hair-like receptors for measurement of several environmental quantities. For sensing of low-frequency flows (typically less than 1 kHz) to obtain information about the environment and avoid, for example, predator attacks, crickets use filiform hairs, which are situated on the dorsal side of two abdominal appendages called cerci and which are able to sense airflows with velocity amplitudes down to 30 m ms$^{-1}$ [3,4] and operate around the energy levels of thermal noise [5]. Crickets gather also information about their environment by the use of bristle hairs, which activate interneurons that respond to tactile stimuli of the cercus and abdomen [6]. Further, crickets have club-shaped sensilla, called clavate hairs, located on their cerci (figure 1), with hair lengths of 20–250 μm [8]. These clavate hairs turn out to be sensitive to (gravitational) acceleration, providing the cricket information on its orientation [7–10]. For example, a cricket uses its clavate hairs to compensate head movement when it is rotated around its longitudinal axis [11], for which such rotations can be measured with a resolution of about 0.1° [9]. Additionally, Bischof [9] showed that these clavate hairs can respond to harmonic accelerations with frequencies up to 300 Hz.

For measuring (gravitational) acceleration, numerous types of accelerometers have been realized over the past years using microelectromechanical systems (MEMS) technology, with applications in, for example, the automotive industry and navigation [12]. Current state-of-the-art commercialized MEMS accelerometers show formidable performance in range, resolution and noise floor. In contrast to the cricket’s clavate system, MEMS accelerometers are usually not hair-based systems and frequently contain feedback electronics. To explore some of the intricacies of the clavate hair system and assess its potential for engineering applications (e.g. automotive industry, robotics and motion tracking), we aim to design, fabricate and characterize a biomimetic accelerometer. Biomimetic hair-based structures have been exploited earlier with applications in both actuation and sensing of physical quantities [13,14], but seldom for inertial measurement. Previously, a hair-like accelerometer has been investigated by Tang et al. [15], but its response to external accelerations was not demonstrated.
In this paper, we present the design and fabrication of a biomimetic accelerometer using MEMS technology and report on its characterization with respect to externally applied accelerations. We also assess, both theoretically and experimentally, responsivity due to flow-induced contributions to the accelerometer’s output and derive design guidelines on how to reduce these effects. Further, we address the accelerometer’s physical limits with respect to noise levels and discuss the possibility for short-term adaptation of the sensor to the environment by electrostatically tuning the sensor’s responsivity.

2. Theory and modelling

2.1. Hair mechanics

Mechanically, the hair-based accelerometer can be understood as a so-called inverted pendulum which is subjected to external accelerations (figure 2). It is described as a second-order rotational-mechanical system with moment of inertia $J$, a rotational stiffness $S$ and a rotational damping $R$, resulting in a description of the system’s response to harmonic accelerations by

$$\theta(\omega) = \frac{T_a}{S - j\omega^2 + j\omega R},$$

(2.1)

where for a cylindrical hair, the moment of inertia can be expressed as

$$J = \frac{\pi \rho d^2}{48} \left( 4L^3 + \frac{3}{4} d^2 L \right).$$

(2.2)

In these expressions, $\theta(\omega)$ is the rotational angle amplitude of the hair and $T_a$ is the amplitude of the torque acting on the hair. Usually, in the models the hairs are treated as cylindrical structures [4], but here the hair is modelled as an inverted conical shape to take the MEMS-hair-shape into account. We will show that this eventually will only lead to a geometry-dependent factor $\eta$ such that $J = \eta \rho d^2 L^3$.

From (2.2), the moment of inertia $J$ depends strongly on the hair diameter $d$ and hair length $L$. The torque $T_a$ is a consequence of external accelerations, or the projections thereof, in the direction perpendicular to the hair and perpendicular to its rotation axis, denoted here by $a_{\text{ext}}$. Using Newton’s second law, $T_a$ is found by integrating the inertial contributions by over the hair

$$T_a = \int_0^d a_{\text{ext}} \rho r\left( \frac{d(z)}{2} \right)^2 dz = \Xi \cdot a_{\text{ext}},$$

(2.3)

where $\Xi$ can be expressed as

$$\Xi = \frac{\pi \rho d^2 L^2}{8}.$$

(2.4)

In these expressions, $\rho$ is the density of the hair and $\eta$ a parameter that depends on the precise geometry of the hair. Further, we assume that the rotational angle amplitudes $\theta$ are small, so the torque $T_a$ can be considered directly proportional to the external acceleration $a_{\text{ext}}$.

2.2. Design

The hair mechanical system behaves like a classical second-order system and consequently exhibits the trade-off between responsivity and bandwidth. The responsivity of the hair accelerometer for frequencies well below the system’s resonance frequency ($\omega < \omega_0$) is defined as

$$\text{responsivity} = \frac{\partial \theta(\omega)}{\partial a_{\text{ext}}} \bigg|_{\omega=0} = \frac{\pi \rho d^2 L^2}{8 S}.$$  

(2.5)

The bandwidth of the system is estimated from the system’s resonance frequency

$$\text{bandwidth} = \omega_0 = \sqrt{\frac{S}{T}}.$$  

(2.6)

By taking the product of responsivity and bandwidth, a figure of merit (FoM) can be defined for the biomimetic hair accelerometer, similar to the approach described by Krijnen et al. [16]

$$\text{FoM} = \text{responsivity} \times \text{bandwidth}.$$  

(2.7)
When the hair length is considerably larger than the hair diameter ($L \gg d$), this FoM can be simplified to

$$\text{FoM} \propto \sqrt{\frac{J^3 L^3}{f^2}}. \quad (2.8)$$

As a result, to achieve a ‘good’ hair-based accelerometer, the sensor should have a long and thick hair (high $d$ and $L$), as well as a compliant mechanical suspension (low $S$).

### 2.3. Flow contributions

Although the hair structure is responsive to external accelerations, dynamic accelerations will lead to a net flow velocity [9]. That is, the hair is moving with a relative velocity $v$ through its surrounding medium, for which we consider the medium velocity itself to be zero

$$v = \frac{a_{\text{ext}}}{\omega}. \quad (2.9)$$

As a consequence, viscous forces will contribute to the total hair mechanical response for external accelerations. Following the approach given by Shimozawa et al. [3], these flow-induced contributions can be taken into account.

The angular deflection amplitude $\theta_{\text{in}}(\omega)$ for a hair subject to external harmonic acceleration at frequency $\omega$, including flow-induced torque, is found to be

$$\theta_{\text{in}}(\omega) = \frac{a_{\text{ext}} \sqrt{\Xi^2 + (\chi / \omega)^2}}{\sqrt{[S - (f + J_{\mu} + J_{\nu})\omega^2]^2 + ([R + R_{\mu})\omega^2]^2}}. \quad (2.10)$$

Here, $\Xi$ is the inertial-induced torque contribution as defined in (2.4), and $\chi$ represents the flow-induced torque contribution (see appendix A)

$$\chi = |Zs|\sqrt{A^2 + B^2}. \quad (2.11)$$

where the variables $A_i$ and $B_i$ are defined as

$$A_i = \int_0^L \kappa(z) \cos (\xi_z + \eta_z) dz \quad \text{and} \quad B_i = \int_0^L \kappa(z) \sin (\xi_z + \eta_z) dz. \quad (2.12)$$

with $\kappa(z)$ a parameter that depends on the flow-profile

$$\kappa(z) = \sqrt{1 + e^{-2|z|} - 2e^{-|z|} \cos \beta z}. \quad (2.13)$$

The quantities $J_{\mu}$ and $J_{\nu}$ are often referred to as virtual added mass, and $R_{\mu}$ as virtual added damping [17]

$$J_{\mu} = \frac{4 \pi \mu^2 L^3}{12}, \quad J_{\nu} = -\frac{\pi^2 \mu^2 G L^3}{3 g \omega^2} \quad \text{and} \quad R_{\mu} = \frac{4}{3} \pi \mu G L^3, \quad (2.14)$$

where $\mu$ is the air dynamic viscosity, and $\rho$ is the air density. The dimensionless parameters $g$ and $G$ are defined in appendix A. Note that when the flow contributions are small ($\chi / \omega \ll \Xi$, $J_{\nu} + J_{\mu} \ll J$ and $R_{\mu} \ll R$), the expression for the rotational angle $\theta_{\text{in}}(\omega)$ will simplify to (2.1).

To evaluate the analytical model, the values for the parameters of the hair-flow sensory system as given in table 1 are used. In this work, given the mechanical parameters $S$ and $R$ listed in table 1, the hair geometry ($L$ and $d$) is chosen such that the inertial-induced $T_i$ is large with respect to $T_f$ (generally about 1000), so that the hair-based accelerometer is primarily inertially responsive. Also, the torsional stiffness $S$ is chosen such—by proper dimensioning the torsional beams which act as the torsional spring—that the resonance frequency of the sensor lies between 100 and 1000 Hz. The torsional damping $R$ is usually determined by squeezed film damping and needs to be found experimentally. The flow contributions contain a (slight) frequency dependency, and therefore a frequency needs to be chosen to evaluate these contributions. As several experiments (e.g. linearity) were performed at 80 Hz, this frequency was used for evaluation.

### 2.4. Thermal noise

An important characteristic of a sensory system is its signal-to-noise ratio (SNR). Generally, when the SNR is equal
or higher than one, a system may detect the input signal. Hence, besides the responsivity of the sensor, the noise level of the system determines the fundamental detection threshold of the system. To find the detection threshold of our system, we consider the power spectral density due to the Johnson–Nyquist (thermal) white noise in the system [18]

$$T_n^2 = 4k_B T_0 (R + R_m),$$

(2.15)

where $k_B$ is the Boltzmann constant, $T_0$ is the ambient temperature and $T_n^2$ the square of the noise-induced equivalent torque per unit of bandwidth. The mechanical transfer function $H(\omega)$ of the system determines the resulting mechanical response to the noise

$$H(\omega) = \frac{1}{S - (j + j_\mu + j_\rho) \omega^2 + j(R + R_m) \omega}.$$  

(2.16)

The equivalent noise angle $\Theta_n$ is found by integrating the noise power over the full spectrum and taking the square root

$$\Theta_n = \sqrt{\int_0^\infty \frac{T_n^2 |H(\omega)|^2 d\omega}{\omega^2}}.$$  

(2.17)

The detection limit $a_{th}$ is found by dividing the sensor's equivalent noise angle $\Theta_n$ by the rotational angle $\Theta_m$ for an acceleration of 1 m s$^{-2}$

$$a_{th} = \frac{\Theta_n}{\Theta_m|_{\omega=1 \text{m s}^{-2}}}.$$  

(2.18)

Thermal noise calculations for our hair mechanical system indicate a detection threshold for acceleration sensing of about $1.1 \times 10^{-3}$ m s$^{-2}$ for frequencies within a bandwidth of 1 kHz.

3. Fabrication

The fabrication process for the biomimetic accelerometer is based upon the process for cricket-inspired biomimetic hair flow sensors, previously developed in our group [19,20]. A schematic overview of the biomimetic accelerometer with the materials indicated is shown in figure 3a.

The sensor is fabricated on a silicon-on-insulator wafer. A layer of 200 nm stoichiometric Si$_3$N$_4$ is used for covering and protecting the trenches. The device layer contains two electrodes, which are used for capacitive read-out of the acceleration-induced movement. On top of the Si$_3$N$_4$ layer, a sacrificial layer of poly-silicon (1.5 μm) is deposited by LPCVD. The sensor membrane and springs are constructed by depositing and patterning a 1 μm SiRN layer on top of the poly-silicon. Aluminium (80 nm) is sputtered on top of the membrane to create the electrodes for capacitive read-out. Our artificial clavate hair is created by two layers of SU-8, to realize both the centre of mass towards the top of the hair structure and a total hair length of about 800 μm with an average diameter of about 80 μm. Finally, to release the membrane the sacrificial poly-silicon layer is removed using XeF$_2$ etching. The fabrication results are shown by the SEM image in figure 3b.

4. Material and methods

4.1. Experimental set-up

Experiments to measure the sensor’s response are performed using the set-up shown in figure 4. A waveform generator (Agilent 33220A-001) is used to produce a sinusoidal signal at a frequency $f_0 = 2 \pi \nu$ that is supplied to an amplifier. This amplifier drives a voice-coil shaker (MB Electronics PM 50) to generate harmonic acceleration. Carrier signals at 1 MHz are supplied to the bottom electrodes of the accelerometer for capacitive read-out. The top electrode is connected to a charge amplifier, and after demodulation and filtering (Stanford SR 650) the sensor’s response is monitored on an oscilloscope (Agilent DSO2064). For calibration, the applied accelerations were measured by mounting a reference accelerometer (STEVIAL-MKI021V1) on the shaker.

4.2. Frequency response

First, the frequency response of the hair-based accelerometer was measured using capacitive read-out in the direction perpendicular to the rotational axis. Frequencies within a range of 50–1000 Hz were applied to the shaker. The reference accelerometer was used to determine the externally applied acceleration amplitude. The resulting measured magnitude response of the biomimetic accelerometer is shown in red in figure 5. Here, the circles represent the measurements and the dashed line exhibits the analytical model based on (2.1), where the resonance frequency $\omega_0$ and the quality factor $Q$ were fitted. We observe good agreement between model and measurements, where the resonance frequency is found to be about 320 Hz.
Additionally, flow measurements have been performed by replacing the shaker by a loudspeaker to apply an oscillating airflow. The measured response for accelerations induced flow is shown in blue in figure 5, where the measured flow response per metre per second has been normalized to acceleration equivalent flow response using (2.9). Here, the squares represent the measurements and the dash-dotted line exhibits the analytical model based on (2.10) with $J = 0$. Again, good agreement between model and measurements is observed, with a similar resonance behaviour. Furthermore, the overall responsivity to acceleration-induced flow is significantly lower than the responsivity to acceleration, as desired.

In addition to the magnitude of the response of the hair-based accelerometer, the phase of its response was also measured. Both measurements (points) and analytical model (dashed line) are shown in figure 6. We observe good agreement between measurements and model, and the phase-shift of about $180^\circ$ around resonance is clearly visible.

### 4.3. Directivity

The sensor’s directivity was measured by rotating it over $360^\circ$, with steps of $10^\circ$, with respect to the direction of the applied external acceleration, while using capacitive read-out. To this end, harmonic acceleration with a frequency of 80 Hz was applied, and the output voltage was measured by a multimeter (Keithley 2000). The obtained results are shown in figure 7.

### 4.4. Threshold and linearity

To obtain a model that describes the sensor’s SNR as a function of acceleration amplitude as well as the sensor’s detection threshold, the signal and noise powers are considered. The signal is assumed to have a linear relationship with respect to the acceleration amplitude $a_0$, given by the coefficient $S_c(\omega)$. This coefficient is directly related to the sensor’s rotational angle $\theta(\omega)$ and therefore has a dependency on the acceleration frequency $\omega$. $N_c$ is a constant representing the total noise, giving for the measured output $z$ due to the uncorrelated sources

$$z(\omega) = \sqrt{(S_c(\omega) a_0)^2 + N_c^2}. \quad (4.1)$$

From this expression, the measured detection threshold $a_{\text{min}}(\omega)$ can be found by equating the noise and signal powers (or alternatively by calculating the intersection value of the asymptotes) which requires only the values for $S_c$ and $N_c$

$$a_{\text{min}}(\omega) = \frac{N_c}{S_c(\omega)}. \quad (4.2)$$

We observe that the measurements are in close agreement with the theoretical response for a so-called figure of eight. The measurements indicate that the hair-based accelerometer has a maximum responsivity for both $0^\circ$ and $180^\circ$, which coincides with the direction perpendicular to the rotational axis of the hair sensor.
Experiments to determine the sensor’s linearity were performed by choosing first a specific acceleration frequency (80 Hz) and then by varying the acceleration amplitude. Subsequently, from the measured output RMS-voltage the sensor’s detection limit and linearity are derived. The results are shown in figure 8, where the points represent the measurements, the solid line is based on (4.1), and the dashed lines indicate the constant equivalent noise amplitude and ideal linear response asymptotes. We observe that for accelerations with an amplitude of more than 0.1 m s$^{-2}$, indicated by the intersection of the asymptotes, the hair-based accelerometer exhibits a clear linear relationship with the applied acceleration. Below this amplitude, the sensor’s output is dominated by noise (SNR $\leq 1$).

To get some insight in the accelerometer’s noise performance and stability, an Allan variance measurement was performed. The zero-acceleration output RMS-voltage was measured with a time interval of 20 ms for a period of 2 h using a multimeter (Agilent 34401A) connected to LabVIEW. The results of the subsequently calculated Allan deviation are shown in figure 9, together with asymptotic lines for both the velocity random walk and the bias instability.

From the linearity measurements, the error on full-scale (i.e. the measurement taken at highest acceleration of 6.12 m s$^{-2}$; figure 8) was calculated and found to be 3.3%. By considering the detection threshold and the full-scale acceleration amplitude, the dynamic range of the hair-based accelerometer is about 35.6 dB. The Allan variance results showed a velocity random walk of 1.67 m s$^{-1}$. The measured threshold of the biomimetic accelerometer is about 0.10 m s$^{-2}$, indicated by the intersection of the asymptotes, the hair-based accelerometer exhibits a clear linear relationship with the applied acceleration. Below this amplitude, the sensor’s output is dominated by noise (SNR $\leq 1$).

To summarize, an overview of the sensor performance is shown in table 2.

5. Discussion

The measured threshold of the biomimetic accelerometer is about 0.10 m s$^{-2}$, which is about a factor 100 larger than the calculated thermal noise based value of $1.1 \times 10^{-3}$ m s$^{-2}$. The relative large detection threshold is due to the presence of comparatively strong noise sources in the read-out electronics [21]. Improvement of these electronics, by reduction of its noise, will help to bring the detection threshold for the accelerometer towards the thermal noise-limited threshold. Similar differences between measured and (calculated) thermal noise levels have been observed for our previously reported biomimetic hair flow sensors [21,22].

As stated in §2, the hair sensor is designed in such a way to have a negligible flow-induced response. Therefore, the ratio between responsivity to acceleration and responsivity to acceleration-induced flow is desired to be large. This ratio is shown in figure 10, and the various graphs indicate its dependence on $L$ and $d$. As stated earlier, the hair length $L$ and diameter $d$ have a large impact on both inertial and flow-induced sensing, and can be used to reduce the sensor’s flow responsivity relative to its acceleration responsivity. In figure 10, the dashed red line indicates the original response ratio, together with the measurements based upon data from figure 5 represented by

![Figure 7](http://rsif.royalsocietypublishing.org/)

**Figure 7.** Measured directivity of the hair-based accelerometer using capacitive read-out at an acceleration frequency of 80 Hz. (Online version in colour.)

![Figure 8](http://rsif.royalsocietypublishing.org/)

**Figure 8.** Measured response versus acceleration amplitudes at a frequency of 80 Hz using capacitive read-out. (Online version in colour.)

![Figure 9](http://rsif.royalsocietypublishing.org/)

**Figure 9.** Measured Allan deviation using capacitive read-out. (Online version in colour.)

**Table 2.** Experimental values of the biomimetic hair-based accelerometer.

<table>
<thead>
<tr>
<th>quantity</th>
<th>symbol</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>mechanical parameters</td>
<td></td>
<td></td>
</tr>
<tr>
<td>quality factor</td>
<td>$Q$</td>
<td>12.5</td>
</tr>
<tr>
<td>resonance frequency</td>
<td>$f_0$</td>
<td>319.6 Hz</td>
</tr>
<tr>
<td>sensor performance (80 Hz)</td>
<td>$\alpha_{\text{ff}}$</td>
<td>$0.10 m s^{-2}$</td>
</tr>
<tr>
<td>full-scale error</td>
<td>$\varepsilon_{\text{FF}}$</td>
<td>3.3%</td>
</tr>
<tr>
<td>velocity random walk</td>
<td>VRW</td>
<td>$1.67 m s^{-1}$</td>
</tr>
<tr>
<td>bias instability</td>
<td>$\alpha_b$</td>
<td>$5 \times 10^{-3} m s^{-2}$</td>
</tr>
<tr>
<td>thermal noise limit</td>
<td>$\alpha_{\text{th}}$</td>
<td>$1.1 \times 10^{-3} m s^{-2}$</td>
</tr>
</tbody>
</table>

$\sqrt{h^{-1}}$ and a bias instability of $5 \times 10^{-3} m s^{-2}$. To summarize, an overview of the sensor performance is shown in table 2.
circles. Although the hair length $L$ has significant impact on both flow and inertial responsivity, only a slight improvement is gained in response ratio for lower frequencies when shortening the hair (dashed-dotted blue line). However, by maintaining the hair length and increasing the thickness of the hair by a factor 2, an improvement of more than 300% can be gained for all frequencies within the sensor’s bandwidth (dotted green line). Also, by considering the FoM for a hair flow sensor [16], the $\text{FoM}_{\text{flow}}$ scales with $d^{1/3}$, while the $\text{FoM}_{\text{acc}}$ of the accelerometer scales with $d^2$ following (2.8). Therefore, to realize a ‘good’ accelerometer, the hair diameter is important for having both a good FoM and minimal flow responsivity. This result is in coincidence with observations for the cricket, as the cricket uses long and thin (filiform) hairs for measurement of flow [23] and shorter and thick (clavate) hairs for measurement of acceleration [8]. Additionally, Sakaguchi & Murphey [10] have shown that crickets use the many clavate hair-sensors on their cerci for determination of their orientation relative to the gravitational field and that they do so both with respect to roll (rotation around longitudinal axis of the animal) and pitch.

Continuing with biology-inspired systems, the performance of a sensory system can be adapted to the environment. For example, the bullfrog’s internal ear displays adaptation by mechanical relaxation of hair bundles [25], leading to a change in stiffness of the saccular hair cell. A similar technique can be applied to the biomimetic accelerometer by electrostatically reducing the torsional stiffness $S$ and could be useful for, for example, robotic applications in which adaptation to the environment is required. The technique of electrostatic spring softening (ESS) has been investigated thoroughly for our biomimetic hair-based flow sensors and can lead to an improvement in responsivity of more than 80% and a reduced sensory threshold of more than 50% in the case of noise which is dominated by, for example, electronics [22]. As the accelerometer’s geometry and parameters are similar to those of the hair flow sensor, comparable results can be expected.

6. Conclusion

A biomimetic accelerometer has been developed and fabricated using surface micromachining and SU-8 lithography, inspired by the clavate hair system of the cricket. We showed that this MEMS hair-based accelerometer has a resonance frequency of 320 Hz, a detection threshold of 0.10 m s$^{-2}$ and a dynamic range of more than 35 dB. Further, the accelerometer has low responsivity to airflow, clear directivity and a bias instability of 5 × 10$^{-3}$ m s$^{-2}$. We have argued that the accelerometer can be further optimized by increasing the hair diameter, and that it principally allows for adaptation to the environment by exploiting the mechanism of ESS.

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Appendix A. Flow contributions

Assuming that the flow with relative velocity $v(t)$ is oscillating over a flat surface, the no-slip boundary condition gives rise to the height $z$-dependent velocity profile [26]

$$v(t) = V_0 \sin(\omega t) - V_0 e^{-\beta z} \sin(\omega t - \beta z),$$

(A 1)
where $\beta$ is proportional to the reciprocal of the boundary layer thickness, with $\nu$ the kinematic viscosity ($\beta = \sqrt{\nu/(2\nu)}$). Using trigonometric identities, expression (A.1) is written as a sinusoidal function with an amplitude $V_z$ and phase-shift $\zeta_z$

$$v_x(t) = V_z \sin (\omega t + \zeta_z),$$  \hspace{1cm} (A.2)

where

$$V_z = V_0 \sqrt{1 + e^{-2\beta z} - 2e^{-\beta z} \cos (\beta z)}$$  \hspace{1cm} (A.3)

and

$$\zeta_z = \arctan \left( \frac{e^{-\beta z} \sin (\beta z)}{1 - e^{-\beta z} \cos (\beta z)} \right).$$  \hspace{1cm} (A.4)

With the velocity profile given, the viscous forces exerted on the hair are described in [27], under the assumption of small angular displacements and low Reynolds and Strouhal numbers [4]. Following the analysis of Shimozawa et al. [3], the torque $T_h$ due to the relative air-movement, acting upon the hair can be expressed as

$$T_h = \sqrt{A^2 + B^2},$$  \hspace{1cm} (A.5)

where $A$ and $B$ are torque contributions given by

$$A = \int_0^L |Z_s| V_z \cos (\zeta_z + \eta_h) dz$$  \hspace{1cm} and

$$B = \int_0^L |Z_s| V_z \sin (\zeta_z + \eta_h) dz.$$  \hspace{1cm} (A 6)

In both $A$ and $B$, the parameter $Z_s$ expresses the relation between the oscillating airflow $V$ and the force per unit length $F$

$$Z_s = \frac{F}{V} = Z_{sr} + jZ_{sz},$$  \hspace{1cm} (A 7)

where

$$Z_{sr} = 4\pi mg$$  \hspace{1cm} and

$$Z_{sz} = \frac{\pi F \omega}{4} - \frac{\omega^2 \mu G}{g}.$$  \hspace{1cm} (A 8)

In these expressions, $G$ and $g$ are dimensionless parameters

$$G = \frac{-g}{g^2 + (\pi/4)^2}, \hspace{1cm} g = \gamma + \ln (s) \hspace{1cm} \text{and} \hspace{1cm} s = \frac{d}{\sqrt{V/\nu}}.$$  \hspace{1cm} (A 9)

References


